Quiz 17

April 5, 2017

Show all work and circle your final answer.

1. Find the Taylor series for $f(x) = 5x^3 - x^2 - 8$ centered at

(a) $a = 0$

f(x) is already a polynomial, so it is it's own Maclaurin series: $\boxed{5x^3 - x^2 - 8}$

(b) $a = -1$

n	$f^{(n)}(x)$	$f^{(n)}(-1)$
0	$5x^3 - x^2 - 8$	$5(-1) - 1 - 8 = -14$
1	$15x^2 - 2x$	$15 - 2(-1) = 17$
2	$30x - 2$	$30(-1) - 2 = -32$
3	30	30
4	0	0
$\vdots$	0	0

So $\boxed{-14 + 17(x+1) + \frac{-32}{2!}(x+1)^2 + \frac{30}{3!}(x+1)^3}$

$= \boxed{-14 + 17(x+1) - 16(x+1)^2 + 5(x+1)^3}$

2. Use the Maclaurin series for $\ln(1+x)$:

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$$

to find a Maclaurin series for $\ln\left(\frac{1+x}{2-2x}\right)$. Write your answer as a constant plus a single series.

$$\ln\left(\frac{1+x}{2-2x}\right) = \ln(1+x) - \ln(2-2x)$$

$$= \ln(1+x) - \ln(2(1-x))$$

$$= \ln(1+x) - \ln 2 - \ln(1-x)$$

$$= \left(\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\right) - \ln 2 - \left(\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(-x)^n}{n}\right)$$

$$= \boxed{-\ln 2 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(x^n - (-x)^n\right)}$$

$$= -\ln 2 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\underbrace{\left(x^n + (-1)^{n+1}x^n\right)}$$

$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ 2x^n & \text{if } n \text{ is odd} \end{cases}$$

$$= \boxed{-\ln 2 + \sum_{n=0}^{\infty}\frac{2x^{2n+1}}{2n+1}}$$